Compact high order schemes for the Helmholtz equation with variable coefficients

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Abstract:

In many problems, one wishes to simulate the propagation of waves in inhomogeneous media, and use a high order accurate method (e.g., fourth order accurate) to alleviate the points-per-wavelength constraint via reducing the dispersion errors. At the same time, one often prefers to have a scheme built on a compact stencil, as it considerably simplifies setting the boundary conditions and also helps reduce the bandwidth of the resulting matrix. The time-harmonic waves in an inhomogeneous medium are governed by the Helmholtz equation with variable coefficients within the Laplacian-like term; besides, the variation of coefficients in the equation may also be due to the geometry. This renders existing fourth order finite difference methods inapplicable. We develop a new compact scheme that is provably fourth order accurate even for these formulations. For exterior problems, one also needs an absorbing boundary condition at the outer surface to mimic the Sommerfeld radiation condition. We construct methods that accomplish this with higher order accuracy as well. Finally, we present numerical results that corroborate the fourth order convergence rate for several model problems.

Joint work with S. Britt (NC State University) and E. Turkel (Tel Aviv University).